Title: Modeling Genetic Drift in Geographically Isolated Wildlife Populations

Brief Overview:

Statistical modeling has proven invaluable to predicting dynamics of wildlife populations. Anthropogenic alteration of ecosystems often results in geographic isolation of subpopulations, which can result in genetic drift that endangers the smaller population. A mathematical model can simulate the effect of such an isolating event. This activity is to be implemented with the Probability Simulator application on the TI-83 graphing calculator. However, it is easily adapted to manipulatives such as dice, spinners, and coins.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*

- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*

- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*

- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*

- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*

- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

  Data Analysis and Probability
  Students will use simulations to make statistical inferences from data to estimate the probability of an event. They will make informed decisions and predictions based upon the results of simulations and data from research.

- **Process Standards**

  Problem Solving
  Students will use the TI-83 Plus graphing calculator equipped with the Probability Simulator Program to simulate the above-described scenario, and to collect and graph data.

Reasoning

Students will interpret and analyze collected data and make predictions.

Communication

Students will use correct mathematical language to explain outcomes of the model and the significance of those outcomes. They will write about mathematical findings.

Links to Maryland High School Mathematics Core Learning Units:

Data Analysis and Probability

- **3.1.3**
  Students will use simulations to make statistical inferences from data to estimate the probability of an event.

- **3.2.1**
  Students will make informed decisions and predictions based upon the results of simulations and data from research.

Links to National Science Education Standards:

- **Life Science**
  Students will observe populations and ecosystems, biological evolution, and interdependence of organisms in conjunction with mathematical modeling of an isolated wildlife population.

- **Science in Personal and Social Perspectives**
  Students will understand environmental quality and natural and human-induced hazards.

Grade/Level:

Grades 9-12, Algebra I & II, Biology

Duration/Length:

Depending on the extent to which the TI-83 Plus is applied to the unit, or whether the unit is taught exclusively with manipulatives, the unit should take between three and four classroom periods and one night of homework to prepare the group report. Extra time may be necessary to instruct some students in the use of the calculator's simulation menu and random number generator.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Computation of percentages
- Use of the TI-83 Plus Probability Simulator (see Appendix1)
- Use of the TI-83 Plus random number generator(see Appendix1)

- Knowledge of Mendelian genetics

Student Outcomes:

Students will:

- learn to use simulations to model natural and anthropogenic phenomena.
- learn about applications of statistics, modeling, and simulations to understanding population genetics, dynamics, and genetic drift.

Materials/Resources/Printed Materials:

- TI-83 Plus graphing calculator with Probability Simulator Application
- Two boxes (shoeboxes) and a large number of small strips of paper
- A computer with spreadsheet software for recording data

Development/Procedures:

A rare ground squirrel population is geographically isolated from the larger population by a highway. These squirrels have three alleles for their fur color. Phenotypically, the squirrels are black, gray and white. Genotypically, there is complete dominance: black* (B) *is dominant over gray* (g) *and white* (w)*, and gray is dominant over white. The allele that produces the white phenotype is actually the absence of pigment, and it is lethal. White individuals do not reproduce.

1. Seeding the population: Use the TI-83 Probability Simulator (follow the directions in Appendix 1) throw a 12-sided die 100 times. Enter the data into Table 1, using the genotype and gender assignments listed in Appendix 1. This is your initial squirrel population.
2. Eliminate all individuals with the lethal genotype **(w,w)**, and record the number of these individuals at the bottom of Table 1.**(Note: these are individuals whose die value was 3 or 9.)**
3. Using the calculator's random integer generator (follow the directions in Appendix 1), select 40% of the individuals by number and eliminate them from the initial pool. This represents attrition due to predation, disease, injury, *etc*.
4. Record the surviving individuals in Table 2, assigning them new individuals numbers so that they are listed sequentially. In other words, so that there are no gaps in the numbers.
5. Write the individual numbers on the strips of paper, segregating the numbers representing males and females into the two boxes.
6. Draw one number from each box and make pairs with each match. These represent the population's mating pairs. Draw numbers until all are matched. If there is a number left over, then retain the individual within the population.
7. Use the probability simulator to throw three coins four times for each mating pair, simulating a litter of four offspring. The first coin represents the genetic trait inherited from the male, the second from the female, and the third coin represents

gender. For the genetic traits: heads is the dominant allele, tails is recessive. For gender, heads is male and tails is female. If the one or both parents is homozygous, then disregard the results of the toss of the corresponding coin, since only one allele can be inherited from that parent. The coin representing gender, however, should never be disregarded. Record the results for each mating pair in a copy of Table A **(note: make many copies of Table A, as you will need one table for every mating pair)**.

8. Record the individuals who survived from the previous generation along with the newly recruited individuals (offspring) sequentially in Table 3.
9. Eliminate individuals that are homozygous for the lethal allele, and record the number of these individuals at the bottom of the table.
10. Repeat Step 3, and record survivors sequentially in Table 4. These individuals are the “second generation” of the population (although in strict terms, this is not exactly true).
11. Repeat this process three more times, recording all five “generations” in the five sets of tables that have been provided, and the mating pair results in as many copies of Table A as is necessary.
12. Use the calculator's random number generator to select 10% of the fifth “generation” population. These individuals represent the subpopulation that is geographically isolated from the larger population.
13. Follow the procedure described above to match mating pairs and to simulate mortality in succeeding “generations,” until the subpopulation is no longer viable; that is, until there are no more surviving offspring or there are no more mating pairs. Record the subpopulation’s “generations” in copies of Tables B & C, making as many copies as is needed, and each mating pair on copies of Table A.

Assessment:

The students should produce a written report as if writing an experiment, and should use histograms of the population dynamics to illustrate the findings of the experiment (see Appendix 2). The histograms should illustrate for each generation (the independent variable) the total surviving population, the genotypic and gender distribution, and the total number of individuals that do not survive due to the lethal allele. The teacher could also provide the students with a set of questions to be answered in the report conclusion, such as the one provided. If the data are collected by groups of students, it is recommended that they write their reports independently. The scoring rubric will be left to the discretion of the teacher.

Extension/Follow Up:

An important extension for this unit would be to discuss actual examples of subpopulations that are endangered by genetic "bottlenecking," such as cheetahs, Florida panthers, *etc.*

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Teacher's Guide:

The process of establishing the initial population and recording the data is laborious, and it is recommended that students work in groups. A thorough knowledge of the calculator programs is essential for the teacher and the students. Spending a class period on the calculator may be necessary. If a die, spinner, and/or coins are used instead of calculator simulations, then the initial population will take at least one class period for two students to complete.

Graphing results is greatly facilitated by using a spreadsheet program. Again, thorough knowledge of the program is required.

Assessment/Answers for the Teacher

The questions below are suggestions, and they are not meant as an exhaustive list. Rather than answering them in the form of a worksheet, it is suggested that they be answered within the conclusion of the laboratory report.

1) How many generations did the isolated population continue before becoming extinct? **- (Should be no more than 10.)**
2) According to your data, did the frequency of the lethal gene increase in the isolated population? **- Yes**. Did this occur in the larger population? **– probably not**
3) Suppose in addition to simulating the isolated population, another student group continued the simulation of the larger population. Would you expect the frequency of the lethal allele to increase, decrease, or remain the same? Justify your answer.**- The frequency of the lethal allele would remain the same because the larger population allows for fewer incidents of "inbreeding" when mating pairs form.**
4) All statistical models attempt to duplicate natural conditions as best as possible, but all are to some degree oversimplifications. What variables were excluded from this model? How might these variables affect the actual outcome? **– Answers may vary**
5) Are there real-life situations that resemble the model? Research one such example of genetic "bottlenecking" that is threatening a small wildlife population. **– Examples: Cheetahs, some African lion populations, the Florida panther, and many Hawaiian species.**

Appendix 1
Table 1: Genotype and Gender Assignments from 12-Sided Die Rolls

Die Value	Genotype	Gender
1	Black homozygous **(BB)**	Male
2	Gray homozygous **(gg)**	Male
3	White homozygous **(ww) lethal**	Male
4	Black heterozygous **(Bg)**	Male
5	Gray heterozygous **(gw) lethal carrier.**	Male
6	Black heterozygous **(Bw) lethal carrier.**	Male
7	Black homozygous **(BB)**	Female
8	Gray homozygous **(gg)**	Female
9	White homozygous **(ww) lethal**	Female
10	Black heterozygous **(Bg)**	Female
11	Gray heterozygous **(gw) lethal carrier.**	Female
12	Black heterozygous **(Bw) lethal carrier.**	Female

Instructions for the Use of the TI-83 Plus Probability Simulator
Seeding the Initial Population

The Probability Simulator is freeware that is available from Texas Instruments. It can be downloaded from a TI website, and by linking to a TI-83 Plus graphing calculator.

1. The simulator is stored in Applications (the **APPS** key). When the Probability Simulator screen appears, press any key to get to the main menu.
2. Select **Roll Dice**.
3. A small menu appears at the bottom of the screen. Select **SET**. This calls up a menu of settings.
4. For the **Trial Set**, key in **100**.
5. Select **one die**.
6. Select **12** sides
7. Select a frequency (**Freq**) graph.
8. Select **All** for the store table (**StoTbl**) item.
9. Clear table (**ClearTbl**) will automatically select **Yes**.
10. Press the **OK** key in the menu at the bottom of the screen
11. Press the **ROLL** key in menu at the bottom of the screen. The data will be displayed as a histogram.

12. Press the **TABL** key in the menu at the bottom of the screen. The data is now displayed as a table that shows roll number, dice value, and sum (which is extraneous since only one die is rolled). The roll number represents individual, the die value corresponds to the genotype and gender assignment in Table 1.

Assigning Offspring Gender and Genotype

1. In the simulator, select **Toss Coins** from the main menu.
2. From the menu at the bottom of the screen, select **SET**.
3. Select **4** for **Trial Set**.
4. Select **3** coins.
5. Select **Freq.**.
6. No other menu items are germane to the simulation, so you can now press OK in the menu on the bottom of the screen.
7. Press **TOSS** in the menu at the bottom of the screen.
8. Select **TABL** from the menu at the bottom of the screen.
9. The **TOSS** number represents the individual offspring.
10. Coin 1 represents the genetic contribution from the male parent, **H** is the dominant gene of the parent, **T** is recessive (if the parent is homozygous, the toss is irrelevant).
11. Coin 2 represents the genetic contribution of the female parent. The rule for **H** and **T** is the same as in step 10.
12. Coin 3 represents gender of the offspring: **H** is male, **T** is female.

Note: The Probability Simulator will only save data while you are in the specific application. Pressing ESC erases all memory within the program.

Instructions for Use of the Random Integer Generator

1. Press the **MATH** key. Move to the Probability **(PRB)** item in the menu at the top of the screen.
2. Select **5:randInt(**.
3. The expression **randInt** appears on the screen. Key in **(1,last # in the population pool)** and press **ENTER.**
4. A number from within the set will appear. The individual corresponding to that number should be removed from the population pool.
5. Repeat step 3 until 40% of the population has been eliminated.

Note: The random number generator will produce duplicate numbers, so it is necessary to keep count of the individuals eliminated from the pool rather than how many numbers are generated.

Appendix 2
Suggested Instructions for the Laboratory Report

Laboratory reports are to be written in grammatically correct and complete sentences. They should be written in the third person, passive tense (*i.e.*, “30 mL of liquid was poured into the beaker”, rather than “We poured 30 mL of liquid into the beaker”). The report should include the following sections.

Abstract

A brief summary of the experiment, the hypothesis, and its outcome. It should ***not*** begin with a sentence such as “The purpose of this experiment was….” Answer the questions: what was the experiment, why was it done, and what does it demonstrate. The abstract should be no more than one paragraph.

Introduction

A more expansive description of the theory being tested, and an explanation of the expected results. This section can be several paragraphs, but need not be too long.

Procedure

A concise but thorough description of the materials and methods used to carry out the experiment. This should include an explanation of the dependent and independent variables.

Results

A verbal explanation of the outcome of the experiment, illustrated with clearly labeled graphs and figures. This is the section that includes the data.

Conclusion

An interpretation of the results that includes an explanation of whether and how the data supports the hypothesis. In addition, a discussion of possible sources of error in the experiment, and what future experiments could address.